Neutrino Factory
∼ An Overview ∼

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Abstract

I make a brief review a current status of neutrino factory. "Neutrino factory" is expected to be a neutrino oscillation experiment of a next generation, which can explore oscillation phenomena with 10^{-4} or 10^{-5} of oscillation probabilities.

1. Introduction

Many experiments and observations have shown evidences for neutrino oscillation one after another. The solar neutrino deficit has long been observed[1, 2, 3, 4, 5]. The atmospheric neutrino anomaly has been found[6, 7, 8, 9] and recently almost confirmed by SuperKamiokande[10]. There is also another suggestion given by LSND[11]. All of them can be understood by neutrino oscillation and hence indicates that neutrinos are massive and there is a mixing in lepton sector[12].

The mixing angles and the mass differences are determined as follows[13],

$$\left\{ \begin{array}{l} \sin^2 2\psi \\ |\delta m^2_{31}| \end{array} \right. \sim \left\{ \begin{array}{l} 1 \\ (1-6)\times 10^{-3}\mathrm{eV}^2 \end{array} \right. \tag{1}$$

by the atmospheric neutrino anomaly and

$$\left\{ \begin{array}{l} \sin^2 2\omega \\ \delta m^2_{21} \end{array} \right. \sim \left\{ \begin{array}{ll} \left\{ \begin{array}{l} 1 \\ (1-10)\times 10^{-5}\mathrm{eV}^2 \end{array} \right. & \text{Large MSW[14]} \\ \left\{ \begin{array}{l} 10^{-3} \\ (1-10)\times 10^{-6}\mathrm{eV}^2 \end{array} \right. & \text{Small MSW} \\ \left\{ \begin{array}{l} 1 \\ (1-1000)\times 10^{-10}\mathrm{eV}^2 \end{array} \right. & \text{Vacuum or LOW Solution} \end{array} \right. \tag{2}$$

by the solar neutrino deficit. Here we assumed three active neutrinos without any sterile one by attributing the solar neutrino deficit and atmospheric neutrino anomaly to the neutrino oscillation and we used the following notation for the mixing matrix U,

$$\begin{aligned} U &= e^{i\psi\lambda_7}\Gamma e^{i\phi\lambda_5}e^{i\omega\lambda_2} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\psi & s_\psi \\ 0 & -s_\psi & c_\psi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{pmatrix} \begin{pmatrix} c_\omega & s_\omega & 0 \\ -s_\omega & c_\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_\phi c_\omega & c_\phi s_\omega & s_\phi \\ -c_\psi s_\omega - s_\psi s_\phi c_\omega e^{i\delta} & c_\psi c_\omega - s_\psi s_\phi s_\omega e^{i\delta} & s_\psi c_\phi e^{i\delta} \\ s_\psi s_\omega - c_\psi s_\phi c_\omega e^{i\delta} & -s_\psi c_\omega - c_\psi s_\phi s_\omega e^{i\delta} & c_\psi c_\phi e^{i\delta} \end{pmatrix}, \end{aligned} \tag{3}$$

where $c_\psi = \cos\psi$, $s_\phi = \sin\phi$, etc.

In next ten years we will have more precise values for the above parameters with the current experiments and the approved experiments[15]. The other parameters,

$$\begin{aligned} \sin\phi &= U_{e3} \\ \sin\delta &: \text{CP phase} , \\ \text{sgn}(\delta m^2_{31}) & \end{aligned} \tag{4}$$

will not, however, be determined in next ten years. To determine them we will need another type of experiments. More to say, with the current upper limit for $\sin\phi$,[16]

$$\sin^2 2\phi < 0.1 \tag{5}$$

we are required to make a precision experiment for neutrino. As a candidate for such a precision measurement, the idea of neutrino factory attracts many physicist. In this review we will see the essence of the neutrino factory[17].

2. Necessity of Neutrino Factory

As we saw in the previous section, we will have to make a precision measurement of neutrino oscillation. The precision of 10^{-4} or better will be required to determine all the physical parameters in lepton sector. It means that we need very well-controlled neutrino beam. Neutrino beam from muon decay can be the best candidate, since it has the following properties.

1. Good energy resolution and well calculated flux.

   We need to know neutrino flux very precisely to see oscillation. Though transition probability may be a small effect, by observing oscillation we will see clearer signal for the oscillation.

2. Availability of incident $\nu_{\rm e}(\bar{\nu}_{\rm e})$ beam.

   T-conjugate channel can be compared in long baseline neutrino oscillation experiments by use of electron neutrinos. It enables to search "matter-effect-free" CP-violation effect[18].

3. "No contamination" in the flux.

   Since there are $\nu_e\bar{\nu}_\mu$ $(\bar{\nu}_e\nu_\mu)$ in the flux, by only observing wrong sign event[17] we see the oscillation.

As indicated in the above, neutrino beam from muon decay seems very promising. In the next section we will review the basic ideas for neutrino factory theoretically.

3. Basic Ideas for Neutrino Factory

3.1. Beam Intensity and Number of Charged Current

In this subsection we see how many charged current we can get with a neutrino factory[19, 20].

First let's estimate how large neutrino flux we can have. In the muon rest-frame, the distributions of neutrinos in the decay of non-polarized muons are given by

$$\begin{aligned} \frac{d^2\sigma_{\nu_\mu,\bar{\nu}_\mu}}{dxdt} &= x^2(3-2x), \\ \frac{d^2\sigma_{\nu_e,\bar{\nu}_e}}{dxdt} &= 6x^2(1-x), \end{aligned} \tag{6}$$

where $x = 2E_\nu/m_\mu$. Then, in the laboratory frame where muons are acceralated, the neutrino fluxes in the direction of muons at the distance L are given by

$$\begin{aligned} \Phi_{\nu_\mu,\bar{\nu}_\mu} &= \gamma^2\frac{n_\mu}{\pi L^2}\left\{2y^2(3-2y)\right\}, \\ \Phi_{\nu_e,\bar{\nu}_e} &= \gamma^2\frac{n_\mu}{\pi L^2}\left\{12y^2(1-y)\right\}, \end{aligned} \tag{7}$$

where n_μ is a number of decaying muons, $\gamma = E_\mu/m_\mu$ and $y = E_\nu/E_\mu$. Note that the total flux increase with E_μ^2.

Next we consider the detection of charged current. The charged current interaction arises as the neutrino-nucleon scattering. Since E_μ is expected to be rather high ($\sim$ tens of GeV) the cross sections for neutrino-nucleon are given by

$$\begin{aligned} \sigma_{\nu N} &\sim 0.67\times 10^{-38}\times E_\nu[\mathrm{GeV}](\mathrm{cm}^2), \\ \sigma_{\bar{\nu} N} &\sim 0.34\times 10^{-38}\times E_\nu[\mathrm{GeV}](\mathrm{cm}^2). \end{aligned} \tag{8}$$

Then from eq.(7) and (8) we can estimate how many charged current we observe in the case of no oscillation with N_k[kt] detector:

$$\begin{aligned} N_{\nu_\mu} &\sim 8\times\frac{n_\mu[10^{21}]E_\mu^3[\text{GeV}]N_k[\text{kt}]}{L^2[1000\text{km}]}, \\ N_{\nu_e} &\sim 7\times\frac{n_\mu[10^{21}]E_\mu^3[\text{GeV}]N_k[\text{kt}]}{L^2[1000\text{km}]}, \\ N_{\bar\nu_\mu} &\sim 4\times\frac{n_\mu[10^{21}]E_\mu^3[\text{GeV}]N_k[\text{kt}]}{L^2[1000\text{km}]}, \\ N_{\bar\nu_e} &\sim 3.5\times\frac{n_\mu[10^{21}]E_\mu^3[\text{GeV}]N_k[\text{kt}]}{L^2[1000\text{km}]}. \end{aligned} \tag{9}$$

Note that the total charged current scales with E_μ^3/L^2.

For example, with $n_\mu = 10^{21}$, $E_\mu = 10\text{GeV}$, $N_k = 10\text{kt}$ and baseline $L = 1000\text{km}$, we will observe $N_\mu = 8\times 10^4$.

On the other hand the number of appearance event, say $\nu_e \to \nu_\mu$, is expected to scale with E_μ, since the transition probability is proportional to L^2/E_ν^2 in the high energy limit and the average of E_ν is almost proportional to E_μ while the number of "parent" neutrino is proportional to E_μ^3/L^2(eq.9):

$$\frac{E_\mu^3}{L^2}\times\frac{L^2}{E_\mu^2}\propto E_\mu. \tag{10}$$

Note that the appearance event increases with E_μ while it does not dependent on L. Of course, though in the actual oscillation experiment, such a simple scaling cannot hold exactly because of initial muon beam distribution, matter effect[14], and so on, the scaling lows (9) and (10) are very helpful for us to understand the feature of a neutrino factory.

3.2. Matter Effect

Neutrinos go through the Earth in the oscillation experiment so the Matter effect,

$$a = 2\sqrt{2}G_F n_e E_\nu, \tag{11}$$

where G_F is Fermion constant and n_e is electron number density, should be considered.

Indeed in matter the effective mixings and effective mass differences in the leading order of $\delta m_{21}^2/\delta m_{31}^2$ for high energy neutrino are modified as follows:

$$\begin{aligned} \psi_m &= \psi, \\ \omega_m &= 0, \end{aligned}$$

$$
\begin{aligned}
\tan 2\phi_m &= \frac{\delta m^2_{31}\sin 2\phi}{\delta m^2_{31}\cos 2\phi \mp a}, \text{ for}\left\{\begin{array}{l}\nu\\ \bar{\nu}\end{array}\right. \qquad (12)\\
\delta m^2_m &= \lambda_+,\ \lambda_-,\ \lambda_+ - \lambda_-\\
& \lambda_\pm = \alpha_\pm \mp \beta_\mp \text{ for}\left\{\begin{array}{l}\nu\\ \bar{\nu}\end{array}\right.\\
& \alpha_\pm = \frac{\delta m^2_{31}\pm a}{2}\\
& \beta_\pm = \frac{1}{2}\sqrt{(\delta m^2_{31}\cos 2\phi \pm a)^2 + (\delta m^2_{31}\sin 2\phi)^2}
\end{aligned}
$$

How strongly the matter effect modify the oscillation probability? Intuitively, since the oscillation length by the matter effect is estimated to be

$$\frac{4E}{a} \sim 3500\text{km} \qquad (13)$$

for the Earth density 3g/cc, it is naively understood that for the baseline longer than 3000 km the matter effect plays a crucial role in the transition probability. In other words we can expand the oscillation part of the transition probabilities in the high energy limit with baseline shorter than 3000km as follows:

$$\sin x \sim x - \frac{1}{3!}x^3. \qquad (14)$$

Even in a very intuitive estimate we should consider the physics of neutrino factory separately depending on the baseline length, $L < 3000$ km and $L >$ 3000km.

4. Physics at Neutrino Factory

In this section we clarify what we can see with neutrino factory and the basic mechanisms. As indicated in (4) there are three quantities which should be measured in the neutrino factory. We study the principle for measuring those quantities[17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. We see it in the order of $\sin\phi$, the sign of δm^2_{31} and CP violation, namely from the easiest to observe to the most difficult to see.

4.1. $\sin\phi$

First we consider U_{e3}. For the shorter baseline ($\leq$ 3000km) the transition probability of $\nu_e \to \nu_\mu$, $P(\nu_e \to \nu_\mu)$ in the high energy limit is given by

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) &= \sin^2\psi \sin^2 2\phi_m \sin^2 \frac{\lambda_+ - \lambda_-}{4E}L \qquad (15)\\
&\simeq \sin^2\psi \sin^2 2\phi \left(\frac{\delta m^2_{31}}{4E}L\right)^2. \qquad (16)
\end{aligned}
$$

Here we used eq.(12) and (14). Indeed it is given by that in vacuum. Then as shown in eq.(10), to see appearance event the higher energy is preferable. More to say, since to avoid systematic error it is better to have higher transition probability, the longer baseline is preferable as long as the above approximation holds.

On the other hand, for the longer length since the approximation given above cannot be satisfied and hence the transition probability is given by

$$P(\nu_e \to \nu_\mu) \quad \Rightarrow \quad \propto \frac{1}{E^2} \sin^2 \frac{aL}{4E}, \tag{17}$$

it is easily understood with eq.(9) that the number of the appearance event decrease with the distance:

$$\text{appearance event} \ \propto \frac{E^3}{L^2} \times \frac{1}{E^2} = \frac{E}{L^2}. \tag{18}$$

From above consideration we can understand intuitively that to get the largest number of appearance event the baseline of a few thousands km and higher energy are most preferable.

This feature can be seen in fig 10 of ref.[21].

4.2. Sign of δm^2_{31} and Matter Effect

Next we consider the sign of δm^2_{31}. In the following $\delta m^2_{31} > 0$ is assumed. If it is negative then neutrino and antineutrino should be exchanged.

As is seen in eq.(12) depending on the sign of δm^2_{31} only one of neutrino species, neutrino or antineutrino, shows the resonance effect. Then intuitively we can expect that by observing which type of neutrino shows the matter resonance we can know the sign of δm^2_{31}. The observation of the sign and that of matter effect is almost same thing. Therefore first of all, we note that to see matter effect clearly we need the baseline length longer than 3000km (see eq.(13)).

From now on we consider how we can see the matter effect and hence the sign of δm^2_{31}.

At first glance, since at the resonance, where $\delta m^2_{31} \cos 2\phi = a$ is satisfied (eq.(12)), the effective mass square $\lambda_+ - \lambda_-$ for the neutrino (see eq.15) has rather small value, the approximation (16) holds in rather large length. Hence we can expect that the event rate takes constant value against rather long baseline length. On the contrary the effective mass square for the anti neutrino becomes larger and hence the event rate against baseline length falls down rather rapidly. From this argument we can expect that the difference between the transition probabilities for neutrino and antineutrino becomes very large at the distance slightly larger than 3000km namely around 5000km.

This feature can be seen in ref.[25]. To measure the sign of δm^2_{31}, in other words, the matter effect, a larger baseline than that for measuring $\sin\phi$ will be preferable, though even with a baseline of 3000km, it can be measured.

4.3. CP violation

Finally we will argue how to see CP violation which will be the most difficult to observe.

First it should be noted that CP Violation is essentially a 3-generation phenomenon, so we have to see "3 generation" in the experiment. It means we should make an experiment with not too high and not too low energy[30].

What are "high" and "low" energies in an oscillation experiment? There are two energy scales in the oscillation experiment intrinsically

$$E \sim \begin{cases} \delta m^2_{31} L \\ \delta m^2_{21} L \end{cases} \tag{19}$$

Then intuitively, to observe CP violating effect we should make an experiment with this energy region. This feature also can be seen in table 1 of ref.[28] from the view of event rate in neutrino factory. By the above argument the best energy for observing CP violation is estimated as follows.

$$E \sim 2.5\mathrm{GeV}\frac{\delta m^2_{31}}{3\times 10^{-3}\mathrm{eV}^2}\frac{L}{300\mathrm{km}}. \tag{20}$$

Around this energy, CP violating part of the transition probability takes the form,

$$8\frac{\delta m^2_{21}L}{2E}\sin^2\frac{\delta m^2_{31}L}{4E}s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega \tag{21}$$

$$\sim \quad 8\frac{\delta m^2_{21}L}{2E}\left(\frac{\delta m^2_{31}L}{4E}\right)^2 s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega. \tag{22}$$

Secondly it should also be noted that the matter effect gives the fake CP violation. Even if there is no intrinsic CP violation, we will see asymmetry between neutrino and antineutrino due to the matter effect. Then we need to avoid or incorporate it.

To avoid it, we should make an experiment with a baseline shorter than 3000km (see eq.(13)). If we can deal with matter effect very well, then longer baseline, namely 3000km will be the best distance since the "event rate" due to CP violation is expected to be maximum(see eq.(9) and (21).) as long as the baseline length and the neutrino energy satisfy the relation between eq.(20). In the author's opinion, however, at that time matter effect cannot be subtracted very well since matter effect also grows with baseline length if we include all the uncertainty of the experimental and theoretical parameters, so we should make an experiment with shorter baseline (and hence lower energy to satisfy (20)) to see CP violation.

To incorporate it we should see T violation channel, say, the difference between $\nu_e \to \nu_\mu$ and $\nu_\mu \to \nu_e$ since the matter effect gives same modification on the transition probabilities. This is very clean signal for CP violation though it will be very difficult to observe the appearance event, $\nu_\mu \to \nu_e$, and hence this difference is very difficult to see experimentally.

5. Conclusion and Discussion

As we have seen in this brief review, by neutrino factory we may explore all the rest parameters for neutrino.

However since such analysis has just started, what is the best experimental setup, namely baseline length, energy and so on, is not fully studied. Only a few parameters are assumed to be unknown in the current analysis. Within such analyses, intuitive argument given in this review holds, especially for determining $\sin\phi$ and sign of δm^2_{31}. We will need to study much more to realize the idea of neutrino factory.

Acknowledgments

This research was supported in part by a Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture, #12047221, #12740157.

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